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A new anomaly-free gauged supergravity in six dimensions

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Abstract: We present a new anomaly-free gauged $\mathcal{N} = 1$ supergravity model in six dimensions. The gauge group is $E_7 \times G_2 \times \mathrm{U}(1)_R$, with all hyperinos transforming in the product representation $(\mathbf{56}, \mathbf{14})$. The theory admits monopole compactifications to $\mathbb{R}^4 \times \mathbf{S}^2$, leading to $D = 4$ effective theories with broken supersymmetry and massless fermions.

Keywords: Field Theories in Higher Dimensions, Anomalies in Field and String Theories, Supergravity Models.

Contents

1. Introduction

Minimal supergravity theories in six dimensions have a remarkably rich structure and have attracted much interest over the years. Some of the reasons motivating the study of such theories are their connection to superstring vacua, their relation to $\mathcal{N} = 2$ theories in $D = 4$ and the framework they provide for cosmological investigations. Among the most interesting models of this type are $D = 6$ gauged supergravity models which have the important property that they spontaneously compactify on lower-dimensional spaces. A prototype for such compactifications is employed in the Salam-Sezgin model [1], a $D = 6$ supersymmetric Einstein-Maxwell theory. This theory admits an $\mathbb{R}^4 \times \mathbf{S}^2$ solution that preserves half the supersymmetries, obtained through a magnetic monopole background residing on $\mathbf{S}^2$ and, as has recently been shown [2], it is the unique maximally-symmetric solution in this class of models.

One further interesting aspect of the monopole compactification in the supersymmetric models is the possibility of making all the U(1) factors in the gauge group massive. In the non-supersymmetric Einstein-Maxwell theory in six dimensions, the monopole compactification gives rise to an effective $D = 4$ chiral gauge theory with gauge group $\mathrm{SU}(2)_{KK} \times \mathrm{U}(1)$. The chirality of the $D = 4$ effective theory is due to the U(1) factor which remains massless and has complex representations. In the supersymmetric generalizations, such as the Salam-Sezgin model or the non-abelian theories discussed in this paper, the vector potentials associated to the U(1) factors of magnetic monopoles acquire a mass due to their Chern-Simons coupling to the second-rank antisymmetric potentials. This coupling is an essential ingredient of all such models.

In addition to the nonzero mass for the U(1) gauge potentials in the monopole directions, most $D = 6$ supergravity theories, including the Salam-Sezgin model, exhibit breakdown of local symmetries due to the presence of gravitational, gauge and mixed anomalies [3] which render such theories inconsistent at the quantum level. In fact, anomaly cancellation has turned out to be a crucial guiding principle for the identification of consistent $D = 6$ theories for the same reason as in the $D = 10$ case. Although the $D = 6$ anomaly cancellation conditions are weaker than those in $D = 10$, they are still very stringent, especially in the case of gauged supergravity theories.

Regarding Poincaré supergravities, there are numerous anomaly-free theories in the literature. Most of these were found by compactifying heterotic string theory on $K3$ using various methods [4] of embedding the $K3$ holonomy group in the SO(32) or $E_8 \times E_8$ gauge group. There are also theories with enhanced symmetry originating from the Gepner points of orbifold realizations of $K3$ [5] or from the non-perturbative mechanism of small instantons [6], as well as theories found by solving the anomaly cancellation conditions alone [7]. Such theories can also be constructed as boundary theories on the six-dimensional orbifold fixed points in seven dimensions compactified on $\mathbf{S}^1 / \mathbb{Z}_2$ [8, 9].

However, in the case of *gauged* supergravities, there is only one known non-trivial anomaly-free model, namely the $E_7 \times E_6 \times \mathrm{U}(1)_R$ model of [10]. This model contains 456 hypermatter fields, identified as a half-hypermultiplet in the pseudoreal **912** of E_7. The theory satisfies a set of highly non-trivial anomaly constraints which make it possible to completely cancel all anomalies by the Green-Schwarz mechanism. Moreover, gauging of $\mathrm{U}(1)_R$ gives rise to a positive-definite potential which implies that one may turn on a magnetic monopole background in a U(1) subgroup of the gauge group and compactify the theory on $\mathbb{R}^4 \times \mathbf{S}^2$. In the particular case considered in [10], where the monopole is embedded in the "hidden" E_6, the fermionic zero modes come exclusively from the E_6 gauginos. The resulting $D = 4$ theory has an $\mathrm{SO}(10) \times \mathrm{U}(1)_R$ gauge symmetry. Unlike the Salam-Sezgin model, supersymmetry is completely broken. For the minimal value of the monopole number, which is required by the stability of the compactification, one obtains two chiral families of SO(10) in the 16-dimensional spinor representation.

One very attractive property of the minimal gauged supergravities in $D = 6$ is that, unlike the superstring theories in $D = 10$, they do not admit the flat spaces as their most symmetric solutions. On the other hand the $\mathbb{R}^4 \times \mathbf{S}^2$ configuration is the unique maximally-symmetric compactification [2]. Furthermore, the expectation values of all the scalars in the model, with the exception of the dilaton, are uniquely determined at the tree level. Hence there is only one modulus accompanying these vacua. Like the $D = 10$ supergravities, these models also admit brane solutions of various dimensions [13, 14, 16, 17, 2]. Because of the uniqueness and simplicity of the $\mathbb{R}^4 \times \mathbf{S}^2$ compactification as well as many shared features with the $D = 10$ heterotic theory it is useful to construct more models of this type and study their low-energy physics.

In this paper, we demonstrate the existence of a new gauged anomaly-free $D = 6$, $\mathcal{N} = 1$ model, besides the known one of [10]. The gauge group here is $E_7 \times G_2 \times \mathrm{U}(1)_R$ and so the theory contains 148 vector multiplets. Restricting to only one tensor multiplet, cancellation of the irreducible gravitational anomaly demands then 392 hypermultiplets. These fit exactly into a half-hypermultiplet of the pseudoreal product representation $(\mathbf{56},\mathbf{14})$ of $E_7 \times G_2$. Again, it is remarkable that all anomalies of the theory cancel in a non-trivial way. Moreover, the model is also free of global anomalies [18, 19] that could potentially arise due to the presence of the G_2 factor. Regarding the bosonic sector, the 1568 real hyperscalars parameterize the quaternionic manifold $\mathrm{Sp}(392,1)/\,\mathrm{Sp}(392) \times \mathrm{Sp}(1)$ and gauging of $\mathrm{U}(1)_R$ contained in $\mathrm{Sp}(1)$ and $E_7 \times G_2$ contained in $\mathrm{Sp}(392)$ yields a positive-definite potential allowing $\mathbb{R}^4 \times \mathbf{S}^2$ compactifications. Unlike the $E_7 \times E_6 \times \mathrm{U}(1)_R$ case, there are fermionic zero modes coming from both the gauginos of the $E_7 \times G_2$ subgroup where the monopole is embedded *and* the hyperinos, since the latter transform non-trivially under both E_7 and G_2 factors. These compactifications generate a rich spectrum of chiral fermions in the $\mathbf{27}$'s (or $\mathbf{16}$'s) of the unbroken E_6 (or $\mathrm{SO}(10)$). However, as we shall show, they are perturbatively unstable. To find an anomaly-free model with a realistic $D = 4$ fermion spectrum still remains a challenging and unsolved problem.

This paper is organized as follows. In section 2 we fix our notation, we describe the basic aspects of gauged $D = 6$ supergravity theories and we write the bosonic lagrangian of our model. In section 3 we explicitly show that the theory is free of anomalies. In section 4 we discuss compactification of the theory on $\mathbb{R}^4 \times \mathbf{S}^2$ and we briefly consider various aspects of the effective $D = 4$ theory. In section 5 we discuss the spectrum of $D = 4$ chiral fermions in detail and show the existence of the tachyonic mode in the spectrum of most compactifications. Finally, in section 6, we summarize and conclude.

2. The model

The building blocks of $D = 6$, $\mathcal{N} = 1$ supergravity theories are the massless representations of the minimal supersymmetry algebra which is chiral and has $\mathrm{Sp}(1)$ as its R-symmetry group. The field content of these representations is summarized in the following multiplets

$$\begin{aligned} \text{Supergravity multiplet}:&\quad (g_{MN}, B^-_{MN}, \psi^{i-}_M)\,,\\ \text{Tensor multiplet}:&\quad (B^+_{MN}, \phi, \chi^{i+})\,,\\ \text{Vector multiplet}:&\quad (A_M, \lambda^{i-})\,,\\ \text{Hypermultiplet}:&\quad (4\varphi, 2\psi^+)\,, \end{aligned} \tag{2.1}$$

where, the $+$ $(-)$ superscripts denote positive (negative) chirality for the spinors and (anti-)self-duality for 2-forms and the index $i = 1,2$ takes values in the fundamental of $\mathrm{Sp}(1)_R$.

A general $D = 6$, $\mathcal{N} = 1$ supergravity theory coupled to matter is constructed by combining one supergravity multiplet with n_T tensor multiplets, n_V vector multiplets and n_H hypermultiplets. Generic string and M-theory compactifications may produce all of these multiplets with arbitrary values of n_T. Anomaly cancellation using the Green-Schwarz mechanism, however, restricts these numbers by the constraint [10]

$$n_H = n_V + 273 - 29 n_T. \tag{2.2}$$

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Starting from the tensor multiplets, we will restrict to the case $n_T = 1$, where there exists a covariant lagrangian description of the model. For this case, the constraint given above reduces to

$$n_H = n_V + 244\,. \tag{2.3}$$

Regarding the hypermultiplets, the $4n_H$ hyperscalars must parameterize a non-compact quaternionic manifold, whose possible forms are given in [11]; here, we will consider the case where the hyperscalar manifold is $\mathrm{Sp}(n_H,1)/\,\mathrm{Sp}(n_H)\times\mathrm{Sp}(1)_R$. Finally, the vector multiplets must belong to a gauge group that is a subgroup of the $\mathrm{Sp}(n_H,1)$ isometry group of the quaternionic manifold, possibly times extra factors under which the hypermultiplets transform as singlets (the E_6 in [10] is one such example). Aside from such factors, one usually chooses the gauge group to be a product of a subgroup of $\mathrm{Sp}(n_H)$ and a subgroup of $\mathrm{Sp}(1)_R$. Under the first factor, the hyperinos may transform in arbitrary representations while the gravitino and tensorino are inert. Under the second factor, the hyperinos are inert (although the hyperscalars are charged) while the gravitino, tensorino and gauginos transform non-trivially. Here, we will consider a $\mathrm{U}(1)_R$ subgroup in which case the gravitino, the tensorino and the gauginos as well as the hyperscalars have all unit charge.

In our model, we pick the gauge group to be $E_7\times G_2\times\mathrm{U}(1)_R$. Thus, we have a total of $133+14+1 = 148$ vector multiplets and eq. (2.3) requires that the total number of hypermultiplets be equal to $148 + 244 = 392$. The hyperinos fit nicely (no singlets!) into a half-hypermultiplet of the pseudoreal 784-dimensional representation $(\mathbf{56},\mathbf{14})$. So, the transformation properties of the various fermions under the three gauge group factors are as follows

$$\begin{aligned}
\psi_M^- &: \ (\mathbf{1},\mathbf{1})_1\,,\\
\chi^+ &: \ (\mathbf{1},\mathbf{1})_1\,,\\
\lambda^- &: \ (\mathbf{133},\mathbf{1})_1+(\mathbf{1},\mathbf{14})_1+(\mathbf{1},\mathbf{1})_1\,,\\
\psi^+ &: \ \frac{1}{2}(\mathbf{56},\mathbf{14})_0\,,
\end{aligned} \tag{2.4}$$

where the subscripts indicate $\mathrm{U}(1)_R$ charges.

Passing on to the hyperscalars, they parameterize the manifold

$$\mathcal{M} = \frac{\mathrm{Sp}(392,1)}{\mathrm{Sp}(392)\times\mathrm{Sp}(1)_R}\,, \tag{2.5}$$

where the holonomy group in the denominator corresponds to the (unbroken) local symmetry of the scalar sector. One may then gauge a subgroup of the isometry group $\mathrm{Sp}(392,1)$; in our case we will consider the gauging of $E_7\times G_2\times\mathrm{U}(1)_R$. The embedding of $E_7\times G_2$ in $\mathrm{Sp}(392)$ is defined by identifying $(\mathbf{56},\mathbf{14})$ with the pseudoreal fundamental representation $\mathbf{784}$ of $\mathrm{Sp}(392)$.

The construction of the gauged supergravity theory proceeds along the steps described in [11, 12]; for the $E_7\times E_6\times\mathrm{U}(1)_R$ case, this procedure was outlined in [10] and discussed in great detail in [16] and can be applied with minor modifications to our model as well.

The only aspects of this construction that need attention refer to the hyperscalar sector. To begin, we let $\alpha = 1, \ldots, 4 \times 392$ label the coordinates on $\mathcal{M}$ and $a = 1, \ldots, 2 \times 392$ label the fundamental of Sp(392). We then pick a gauge-fixed coset representative L and we decompose its Maurer-Cartan form into the coset vielbein and the Sp(392) and $\mathrm{Sp}(1)_R$ connections

$$\begin{aligned} V_\alpha^{ai} &= (L^{-1}\partial_\alpha L)^{ai}\,, \\ \mathcal{A}_\alpha^{ab} &= (L^{-1}\partial_\alpha L)^{ab}\,, \\ \mathcal{A}_\alpha^{ij} &= (L^{-1}\partial_\alpha L)^{ij}\,, \end{aligned} \tag{2.6}$$

whose pullbacks on the spacetime manifold define the composite vielbein and connections

$$\begin{aligned} P_M^{ai} &= (L^{-1}\partial_M L)^{ai} = \partial_M \varphi^\alpha V_\alpha^{ai}\,, \\ Q_M^{ab} &= (L^{-1}\partial_M L)^{ab} = \partial_M \varphi^\alpha\, \mathcal{A}_\alpha^{ab}\,, \\ Q_M^{ij} &= (L^{-1}\partial_M L)^{ij} = \partial_M \varphi^\alpha\, \mathcal{A}_\alpha^{ij}\,, \end{aligned} \tag{2.7}$$

that are used to construct scalar kinetic terms and spinor covariant derivatives respectively. Gauging $E_7 \times G_2 \times \mathrm{U}(1)_R$ entails introducing the E_7 gauge fields A_M^I, $I = 1, \ldots, 133$, the G_2 gauge fields $A_M^{I'}$, $I' = 1, \ldots, 14$ and the $\mathrm{U}(1)_R$ gauge field A_M^3 and replacing ordinary derivatives by gauge-covariant ones. In particular, the covariant derivative acting on the hyperscalars is

$$\mathcal{D}_M\, \varphi^\alpha = \partial_M \varphi^\alpha - g(A_M^I \xi^{I\alpha} + A_M^{I'} \xi^{I'\alpha}) - g' A_M^3 \xi^{3\alpha}\,, \tag{2.8}$$

where T^I, $T^{I'}$ and T^3 are the antihermitian E_7, G_2 and $\mathrm{U}(1)_R$ generators, $\xi^{I\alpha} = (T^I \varphi)^\alpha$, $\xi^{I'\alpha} = (T^{I'}\varphi)^\alpha$ and $\xi^{3\alpha} = (T^3\varphi)^\alpha$ are Killing vectors associated with the respective isometries and g and g' are the $E_7 \times G_2$ and $\mathrm{U}(1)_R$ couplings. Accordingly, the composite vielbein and connections in (2.7) are replaced by the gauged versions

$$\begin{aligned} \mathcal{P}_M^{ai} &= (L^{-1}\, \mathcal{D}_M\, L)^{ai} = \mathcal{D}_M\, \varphi^\alpha V_\alpha^{ai}\,, \\ \mathcal{Q}_M^{ab} &= (L^{-1}\, \mathcal{D}_M\, L)^{ab} = \mathcal{D}_M\, \varphi^\alpha\, \mathcal{A}_\alpha^{ab}\,, \\ \mathcal{Q}_M^{ij} &= (L^{-1}\, \mathcal{D}_M\, L)^{ij} = \mathcal{D}_M\, \varphi^\alpha\, \mathcal{A}_\alpha^{ij} - g' A_M^3 (T^3)^{ij}\,. \end{aligned} \tag{2.9}$$

A direct consequence of the gauging of $\mathrm{U}(1)_R$ is the emergence of a scalar potential in the theory. This comes about due to the fact that the commutator $[\mathcal{D}_M, \mathcal{D}_N]\epsilon^i$ appearing in the supersymmetry variation of the gravitino kinetic term acquires an extra term involving the gauge field strengths and the functions

$$\begin{aligned} C_{ij}^I = (L^{-1} T^I L)_{ij}\,, \quad C_{ij}^{I'} &= (L^{-1} T^{I'} L)_{ij}\,, \\ C_{ij}^3 &= (L^{-1} T^3 L)_{ij}\,. \end{aligned} \tag{2.10}$$

Restoring local supersymmetry requires then a set of modifications to the lagrangian and transformation rules; in the bosonic sector, this induces a hyperscalar potential.

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The lagrangian of the gauged supergravity theory was first derived in [11] and further elaborated upon in [12]. Its bosonic part may be written as

$$
\begin{aligned}
e^{-1}\,\mathcal{L} = \frac{1}{4}R - \frac{1}{4}\partial_M\phi\partial^M\phi - \frac{1}{12}e^{2\phi}G_{MNP}G^{MNP} - \frac{1}{4}e^{\phi}v_A\,\mathrm{tr}(F_{AMN}F_A^{MN}) - \\
-g_{\alpha\beta}(\varphi)\,\mathcal{D}_M\,\varphi^\alpha\,\mathcal{D}^M\,\varphi^\beta + \frac{1}{8}e^{-1}\epsilon^{MNPQRS}B_{MN}v_A\,\mathrm{tr}(F_{APQ}F_{ARS}) - \\
-\frac{1}{4}e^{-\phi}\Big[g^2(v_7^{-1}C^I_{ij}C^{Iij} + v_2^{-1}C^{I'}_{ij}C^{I'ij}) + g'^2 v_1^{-1}C^3_{ij}C^{3ij}\Big]\,. \qquad (2.11)
\end{aligned}
$$

Here, the summation index $A = 7, 2, 1$ runs over the three gauge group factors, v_A are a set of constants to be determined later and the various traces are interpreted as e.g. $\mathrm{tr}(F_{7MN}F_7^{MN}) = F^I_{MN}F^{IMN}$. In this form of the lagrangian, the field strength of B_{MN} is given by the usual definition $G_{MNP} = 3\partial_{[M}B_{NP]}$ and the Green-Schwarz term is explicit.

3. Anomaly cancellation

In this section, we demonstrate that the theory described above is anomaly-free. Generally, $D = 6$ chiral supergravities suffer from gravitational, gauge and mixed anomalies arising from box diagrams with four external gravitons and/or gauge bosons and one chiral spinor or (anti-)self-dual 2-form running in the loop. Starting from the gravitational anomalies, we first note that the contributions from the 2-forms from the supergravity and tensor multiplets cancel each other so that the only nonzero terms come from the gravitino, tensorino, gauginos and hyperinos. Summing these contributions (in that order) using the formulas of the appendix, we find the expression

$$
\begin{aligned}
I_8(R) &= -\left[\frac{49}{72}\,\mathrm{tr}\,R^4 - \frac{43}{288}(\mathrm{tr}\,R^2)^2\right] + (1 - 148 + 392)\left[\frac{1}{360}\,\mathrm{tr}\,R^4 + \frac{1}{288}(\mathrm{tr}\,R^2)^2\right] \\
&= (\mathrm{tr}\,R^2)^2\,, \qquad (3.1)
\end{aligned}
$$

which confirms that the irreducible $\mathrm{tr}\,R^4$ terms cancel and explains the particular normalization chosen. Turning to the gauge anomalies, these may be split into (i) E_7 and G_2 anomalies (contributions from gauginos and hyperinos), (ii) $\mathrm{U}(1)_R$ anomalies (contributions from gravitino, tensorino and gauginos), (iii) $E_7 \times G_2$ anomalies (contributions from hyperinos) and (iv) $E_7 \times \mathrm{U}(1)_R$ and $G_2 \times \mathrm{U}(1)_R$ anomalies (contributions from gauginos). Writing down the various contributions in the order indicated above, we find

$$
\begin{aligned}
I_8(F) &= \frac{2}{3}(-\,\mathrm{Tr}\,F_7^4 + 7\,\mathrm{tr}\,F_7^4) + \frac{2}{3}(-\,\mathrm{Tr}\,F_2^4 + 28\,\mathrm{Tr}\,F_2^4) + \frac{2}{3}(-5 + 1 - 148)F_1^4 + \\
&\quad +2\,\mathrm{tr}\,F_7^2\,\mathrm{Tr}\,F_2^2 - 4\,\mathrm{Tr}\,F_7^2F_1^2 - 4\,\mathrm{Tr}\,F_2^2F_1^2 \\
&= -\frac{2}{3}\,\mathrm{Tr}\,F_7^4 + \frac{14}{3}\,\mathrm{tr}\,F_7^4 + 18\,\mathrm{Tr}\,F_2^4 - \frac{304}{3}F_1^4 + \\
&\quad +2\,\mathrm{tr}\,F_7^2\,\mathrm{Tr}\,F_2^2 - 4\,\mathrm{Tr}\,F_7^2F_1^2 - 4\,\mathrm{Tr}\,F_2^2F_1^2\,. \qquad (3.2)
\end{aligned}
$$

where “tr” and “Tr” stand for fundamental and adjoint traces respectively. Finally, we have to consider the mixed anomalies. Splitting them into (i) mixed E_7 and G_2 anomalies (contributions from gauginos and hyperinos) and (ii) mixed $\mathrm{U}(1)_R$ anomalies (contributions from gravitino, tensorino and gauginos), we write

$$\begin{aligned} I_8(F,R) &= \frac{1}{6}\operatorname{tr} R^2(\operatorname{Tr} F_7^2 - 7\operatorname{tr} F_7^2 + \operatorname{Tr} F_2^2 - 28\operatorname{Tr} F_2^2) + \frac{1}{6}(-19-1+148)\operatorname{tr} R^2 F_1^2 \\ &= \operatorname{tr} R^2\left(\frac{1}{6}\operatorname{Tr} F_7^2 - \frac{7}{6}\operatorname{tr} F_7^2 - \frac{9}{2}\operatorname{Tr} F_2^2\right) + \frac{64}{3}\operatorname{tr} R^2 F_1^2\,. \end{aligned} \tag{3.3}$$

Eqs. (3.2)–(3.3) can be simplified by expressing all traces in the fundamental representations. Moreover, since both E_7 and G_2 factors do not possess fourth-order invariants, all of the above traces can be expressed exclusively in terms of second-order traces. For the two factors, we have the identities

$$\begin{aligned} \operatorname{Tr} F_7^2 &= 3\operatorname{tr} F_7^2\,, & \operatorname{Tr} F_2^2 &= 4\operatorname{tr} F_2^2\,, \\ \operatorname{tr} F_7^4 &= \frac{1}{24}(\operatorname{tr} F_7^2)^2\,, & \operatorname{tr} F_2^4 &= \frac{1}{4}(\operatorname{tr} F_2^2)^2\,, \\ \operatorname{Tr} F_7^4 &= \frac{1}{6}(\operatorname{tr} F_7^2)^2\,, & \operatorname{Tr} F_2^4 &= \frac{5}{2}(\operatorname{tr} F_2^2)^2\,. \end{aligned} \tag{3.4}$$

Substituting these in (3.2) and (3.3), we write the gauge and mixed anomalies in the simplified form

$$I_8(F) = \frac{1}{12}(\operatorname{tr} F_7^2)^2 + 45(\operatorname{tr} F_2^2)^2 - \frac{304}{3}F_1^4 + 8\operatorname{tr} F_7^2\operatorname{tr} F_2^2 - 12\operatorname{tr} F_7^2 F_1^2 - 16\operatorname{tr} F_2^2 F_1^2\,, \tag{3.5}$$

and

$$I_8(F,R) = \operatorname{tr} R^2\left(-\frac{2}{3}\operatorname{tr} F_7^2 - 18\operatorname{tr} F_2^2 + \frac{64}{3}F_1^2\right). \tag{3.6}$$

Putting the three contributions (3.1), (3.5) and (3.6) together, we find that the total anomaly polynomial is given by

$$\begin{aligned} I_8 &= (\operatorname{tr} R^2)^2 + \operatorname{tr} R^2\left(-\frac{2}{3}\operatorname{tr} F_7^2 - 18\operatorname{tr} F_2^2 + \frac{64}{3}F_1^2\right) + \frac{1}{12}(\operatorname{tr} F_7^2)^2 + \\ &\quad +45(\operatorname{tr} F_2^2)^2 - \frac{304}{3}F_1^4 + 8\operatorname{tr} F_7^2\operatorname{tr} F_2^2 - 12\operatorname{tr} F_7^2 F_1^2 - 16\operatorname{tr} F_2^2 F_1^2\,. \end{aligned} \tag{3.7}$$

In order for the Green-Schwarz mechanism to operate, the above polynomial must factorize as

$$I_8 = (\operatorname{tr} R^2 + u_A\operatorname{tr} F_A^2)(\operatorname{tr} R^2 + \tilde{u}_A\operatorname{tr} F_A^2)\,. \tag{3.8}$$

To check whether this is possible, one must (i) compare the $(\operatorname{tr} F_A^2)^2$ and $\operatorname{tr} R^2\operatorname{tr} F_A^2$ terms to determine $(u_A,\tilde{u}_A)$ and (ii) check if, for the values of $(u_A,\tilde{u}_A)$ thus determined, the $\operatorname{tr} F_A^2\operatorname{tr} F_B^2$ cross-terms match as well. Remarkably, it turns out that all conditions are indeed satisfied with

$$\begin{aligned} u_7 &= -\frac{1}{2}\,, & u_2 &= -3\,, & u_1 &= -4\,, \\ \tilde{u}_7 &= -\frac{1}{6}\,, & \tilde{u}_2 &= -15\,, & \tilde{u}_1 &= \frac{76}{3}\,. \end{aligned} \tag{3.9}$$

Cancellation of anomalies is then a straightforward matter. One may set the undetermined constants v_A in (2.11) equal to

$$v_A = -u_A\,, \tag{3.10}$$

and modify the B_{MN} gauge transformation law to

$$\delta B_2 \sim \omega^1_{2L} + \tilde{u}_A \omega^1_{2Y,A}\,, \tag{3.11}$$

where ω^1_{2L} and $\omega^1_{2Y,A}$ are related to $\mathrm{tr}\, R^2$ and $\mathrm{tr}\, F_A^2$ by descent. The corresponding anomalous variation of the Green-Schwarz term in (2.11) (including a gravitational term) cancels exactly the variation of the effective action.

Apart from the perturbative anomalies discussed above, our model may also have global anomalies. In particular, since the G_2 factor in the gauge group has a non-trivial sixth homotopy group, $\pi_6(G_2) = \mathbb{Z}_3$, there exist gauge transformations not continuously connected to the identity. Under such transformations, the effective action may pick up a phase factor and is thus ill-defined. The condition for the absence of global anomalies in the case of the G_2 gauge group is given by [20]

$$1 - 4\sum_{\mathcal{R}} n_{\mathcal{R}} b_{\mathcal{R}} = 0 \mod 3\,, \tag{3.12}$$

where $n_{\mathcal{R}}$ is the number of hypermultiplets transforming in the representation $\mathcal{R}$ of G_2 and $b_{\mathcal{R}}$ is defined by $\mathrm{tr}_{\mathcal{R}}\, F_2^4 = b_{\mathcal{R}} (\mathrm{tr}\, F_2^2)^2$. In our model, the hypermultiplets are in the adjoint of G_2 and we have $n_{\mathbf{14}} = \frac{1}{2} \times 56 = 28$ and $b_{\mathbf{14}} = \frac{5}{2}$. For these numbers, the condition (3.12) is indeed satisfied and thus the theory is free of global anomalies as well.

4. Compactification

The bosonic lagrangian (2.11) contains a hyperscalar potential given by its last term. Employing the definitions $g_7 = g/\sqrt{v_7}$, $g_2 = g/\sqrt{v_2}$ and $g_1 = g'/\sqrt{v_1}$, we rewrite this potential as

$$V(\varphi) = \frac{1}{4} e^{-\phi} (g_7^2 C^I_{ij} C^{Iij} + g_2^2 C^{I'}_{ij} C^{I'ij} + g_1^2 C^3_{ij} C^{3ij})\,, \tag{4.1}$$

where the various C-functions are given in (2.10). In [16], it was shown that a convenient parameterization of the scalar coset is given by the 784×2 matrix

$$\varphi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_{392} \end{pmatrix}, \tag{4.2}$$

where φ_n, $n = 1, \ldots, 392$, are themselves 2×2 matrices satisfying the reality condition $\varphi_n^* = \sigma_2 \varphi_n \sigma_2$. With this parameterization, one may define the coset representative as the $(784 + 2) \times (784 + 2)$ matrix

$$L = \begin{pmatrix} 1 + \left(\frac{\sqrt{1+\varphi^\dagger\varphi} - 1}{\varphi^\dagger\varphi} \right) \varphi\varphi^\dagger & \varphi \\ \varphi^\dagger & \sqrt{1 + \varphi^\dagger\varphi} \end{pmatrix}, \tag{4.3}$$

where the factor inside parentheses is understood as a scalar (since $\varphi^\dagger\varphi$ is proportional to the identity). Using the definition (4.3), it can be shown that the C-functions take the form

$$C^I_{ij} = (\varphi^\dagger T^I \varphi)_{ij}\,, \qquad C^{I'}_{ij} = (\varphi^\dagger T^{I'} \varphi)_{ij}\,,$$
$$C^3_{ij} = \left[1 + \mathrm{tr}(\varphi^\dagger\varphi)\right](T^3)_{ij}\,. \tag{4.4}$$

Then the potential is given by the simple expression

$$V(\varphi) = \frac{1}{16}e^{-\phi}\left[-g_7^2(\varphi^\dagger T^I\varphi)^2 - g_2^2(\varphi^\dagger T^{I'}\varphi)^2\right] + \frac{1}{8}e^{-\phi}g_1^2\left[1 + \mathrm{tr}(\varphi^\dagger\varphi)\right]^2. \tag{4.5}$$

Using this expression and recalling that T^I and $T^{I'}$ are antihermitian, we immediately see that the potential is strictly positive-definite and attains its unique global minimum at $\varphi^\alpha = 0$. Among other things, this implies that the $E_7 \times G_2$ gauge symmetry cannot be spontaneously broken by the hyperscalars at tree level.

At $\varphi^\alpha = 0$, the potential (4.5) takes the exponential form

$$V_{\min} = \frac{1}{8}e^{-\phi}g_1^2\,, \tag{4.6}$$

and, for the case of constant ϕ, it acts like a cosmological constant. It is this effective cosmological constant and the particular form of the dilaton coupling which picks up the $\mathbb{R}^4 \times \mathbf{S}^2$ among other maximally-symmetric spaces [1]. In the non-supersymmetric theory, de Sitter or anti- de Sitter spaces would also be possible solutions [15].

In the $E_7 \times E_6 \times \mathrm{U}(1)_R$ model of [10], the monopole was embedded in the E_6 factor, yielding a $D = 4$ effective theory with $\mathrm{SO}(10) \times \mathrm{U}(1)_R$ gauge symmetry where all massless fermions originate from the E_6 gauginos since only the latter couple to the monopole; in [16] this was generalized to include monopole embeddings in E_7 where zero-mode fermions arise from the hyperinos as well. In the model considered here, one can embed as many magnetic monopoles as the rank of the gauge group which is 10. In general we will thus have a maximum of 10 monopole charges. This will give rise to fermionic zero modes from the associated gauginos but, since the hyperinos are charged under both E_7 and G_2 gauge group factors, it will also necessarily give rise to fermionic zero modes from the associated hyperinos. In the absence of a *vev* for the vector potential associated to $\mathrm{U}(1)_R$, the gravitino, tensorino and the rest of the gauginos will be massive. Turning to the bosons, the squared mass of each one of the lightest hyperscalar fluctuations will receive two contributions, one being proportional to the associated eigenvalue of $\frac{\partial^2 V}{\partial\varphi^\alpha\partial\varphi^\beta}$ at $\varphi^\alpha = 0$ and the other being proportional to D^2 where D is the covariant derivative acting on the hyperscalar fluctuations in the background of the monopole vector potential(s). The first contribution will make all hyperscalars massive. The second contribution, if the monopole charges do not add up to zero, will be a positive quantity proportional to $1/a^2$, where a is the radius of $\mathbf{S}^2$. Furthermore in the case of a nonzero net monopole charge of the hyperscalar even the leading (lightest) $D = 4$ scalar modes resulting from it will belong to a non trivial irreducible representation of the Kaluza-Klein SU(2). We shall comment on the masses of some other bosonic modes in section 6.

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In the absence of a *vev* for the $\mathrm{U}(1)_R$ gauge field, the supersymmetric variation of the gravitino will be nonzero and thus this class of compactifications will break all supersymmetries.

To write down the ansatz for monopole compactification, we employ the rescalings $A_M^I \to A_M^I/g$, $A_M^{I'} \to A_M^{I'}/g$ and $A_M^3 \to A_M^3/g'$ and we set the metric, the $\mathrm{U}(1) \subset E_7$ gauge field and the dilaton equal to

$$\begin{gathered} ds_6^2 = \eta_{\mu\nu} dx^\mu dx^\nu + a^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \\ A_\pm = \frac{n}{2} Q(\cos\theta \mp 1) d\varphi\,; \qquad F = \frac{n}{2} Q \sin\theta d\theta \wedge d\varphi\,, \\ \phi = \phi_0 = \text{const.} \end{gathered} \tag{4.7}$$

Here, A_+ and A_- correspond to the potentials on the northern and southern hemisphere which, on the equator, should be connected by a gauge transformation parameterized by $U = e^{inQ\varphi}$. In order for U to be single-valued as φ changes by 2π, the quantity nq_{min}, where q_{min} is the minimal $\mathrm{U}(1)_M$ charge in the theory, must be an integer. In the above, Q is any generator of the gauge group and, in general, can be a linear combination of all commuting generators with appropriate quantization conditions on the coefficients. This ansatz solves the field equations [10].

To prepare the setting for the spectrum analysis of the next section let us consider some examples. A first example is given by embedding $\mathrm{U}(1)_M$ in E_7 according to the maximal-subgroup decomposition

$$E_7 \supset E_6 \times \mathrm{U}(1)\,. \tag{4.8}$$

Using the branching rules [21]

$$\begin{aligned} \mathbf{56} &\to \mathbf{27}_1 + \overline{\mathbf{27}}_{-1} + \mathbf{1}_3 + \mathbf{1}_{-3}\,, \\ \mathbf{133} &\to \mathbf{78}_0 + \mathbf{27}_{-2} + \overline{\mathbf{27}}_2 + \mathbf{1}_0\,, \end{aligned} \tag{4.9}$$

we see that $q_{\text{min}} = 1$ so that $n =$ integer. Discarding neutral fields, we see that the fermion representations under $E_6 \times G_2 \times \mathrm{U}(1)_M$ which can give rise to fermion zero modes on $\mathbf{S}^2$ are,

$$(\mathbf{27}, \mathbf{14})_1 + (\overline{\mathbf{27}}, \mathbf{14})_{-1} + (\mathbf{1}, \mathbf{14})_3 + (\mathbf{1}, \mathbf{14})_{-3}\,, \tag{4.10}$$

for the E_7 hyperinos and

$$(\mathbf{27}, \mathbf{14})_{-2} + (\overline{\mathbf{27}}, \mathbf{14})_2\,, \tag{4.11}$$

for the E_7 gauginos.

As a second example, consider the successive maximal-subgroup decompositions

$$E_7 \supset \mathrm{SO}(12) \times \mathrm{SU}(2) \supset \mathrm{SO}(10) \times \mathrm{SU}(2) \times \mathrm{U}(1)\,, \tag{4.12}$$

and identify the last U(1) factor with $\mathrm{U}(1)_M$. Using the branching rules

$$\begin{aligned} \mathbf{56} &\to (\mathbf{12}, \mathbf{2}) + (\mathbf{32}, \mathbf{1})\,, \\ \mathbf{133} &\to (\mathbf{1}, \mathbf{3}) + (\mathbf{32}', \mathbf{2}) + (\mathbf{66}, \mathbf{1})\,, \end{aligned} \tag{4.13}$$

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for $E_7 \supset \mathrm{SO}(12) \times \mathrm{SU}(2)$ and

$$\begin{aligned}
\mathbf{12} &\to \mathbf{1}_1 + \mathbf{1}_{-1} + \mathbf{10}_0\,, \\
\mathbf{32} &\to \mathbf{16}_1 + \overline{\mathbf{16}}_{-1}\,, \\
\mathbf{32'} &\to \mathbf{16}_{-1} + \overline{\mathbf{16}}_1\,, \\
\mathbf{66} &\to \mathbf{1}_0 + \mathbf{10}_2 + \mathbf{10}_{-2} + \mathbf{45}_0\,,
\end{aligned} \tag{4.14}$$

for $\mathrm{SO}(12) \supset \mathrm{SO}(10) \times \mathrm{U}(1)$, we see again that, $q_{\min} = 1$ and $n =$ integer. Discarding neutral fields, we find that the muliplets of $\mathrm{SO}(10) \times \mathrm{SU}(2) \times G_2 \times \mathrm{U}(1)_M$ which can have fermion zero modes on $\mathbf{S}^2$ are

$$(\mathbf{1},\mathbf{2},\mathbf{14})_1 + (\mathbf{1},\mathbf{2},\mathbf{14})_{-1} + (\mathbf{16},\mathbf{1},\mathbf{14})_1 + (\overline{\mathbf{16}},\mathbf{1},\mathbf{14})_{-1}\,, \tag{4.15}$$

for the hyperinos and

$$(\mathbf{16},\mathbf{2},\mathbf{14})_{-1} + (\overline{\mathbf{16}},\mathbf{2},\mathbf{14})_1 + (\mathbf{10},\mathbf{1},\mathbf{14})_2 + (\mathbf{10},\mathbf{1},\mathbf{14})_{-2}\,, \tag{4.16}$$

for the E_7 gauginos. In the above examples, the representations (4.10) and (4.15) are understood as half-hypermultiplets.

5. The chiral spectrum and (in-)stabilities

The monopole embeddings discussed in the previous section give rise to many chiral fermions in the complex representations of the unbroken gauge group in the effective $D = 4$ theory. Using the formalism of [15] we can evaluate the content of the effective $D = 4$ theory. Here we apply this formalism to the two examples of the previous section.

In the first example the unbroken gauge group is $E_6 \times G_2 \times \mathrm{U}(1)_R \times \mathrm{SU}(2)_{KK}$, where $\mathrm{SU}(2)_{KK}$ denotes the Kaluza-Klein gauge group originating from the isometries of $\mathbf{S}^2$. The chiral fermions originate from the $\mathbf{27}$'s and the $\overline{\mathbf{27}}$'s. We can regard all the $D = 4$ fermions as left-handed Weyl spinors. The chiral fermions originating from the decomposition of $\mathbf{56}$ of E_7 then are

$$2(\overline{\mathbf{27}},\mathbf{14},\mathbf{n})_0\,, \tag{5.1}$$

while the fermions originating from the decomposition of the adjoint of E_7 produce

$$(\mathbf{27},\mathbf{14},\mathbf{2n})_1 + (\mathbf{27},\mathbf{14},\mathbf{2n})_{-1}\,, \tag{5.2}$$

with the subscripts here denoting the $\mathrm{U}(1)_R$ charges.

In the second example the unbroken gauge group in $D = 4$ is $G = \mathrm{SO}(10) \times \mathrm{SU}(2) \times G_2 \times \mathrm{SU}(2)_{KK} \times \mathrm{U}(1)_R$. The spectrum of the $D = 4$ chiral fermions is given by

$$2(\overline{\mathbf{16}},\mathbf{1},\mathbf{14},\mathbf{n})_0\,, \tag{5.3}$$

and

$$(\mathbf{16},\mathbf{2},\mathbf{14},\mathbf{n})_1 + (\mathbf{16},\mathbf{2},\mathbf{14},\mathbf{n})_{-1}\,. \tag{5.4}$$

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It is clearly seen that the spectrum in both cases is free from all chiral anomalies, because E_6 and SO(10) are safe groups in $D = 4$ and the $\mathrm{U}(1)_R$ couplings are obviously vectorlike. It is also seen that there is no value of n which produces a realistic spectrum. One can study other embeddings with the aim of reducing the gauge group and the number of families. For example, the group G_2 can be broken completely by the embedding of a monopole in an SU(2) subgroup of G_2 relative to which the branching is $\mathbf{14} = \mathbf{3} + \mathbf{11}$. By itself this will produce only a vectorlike theory in $D = 4$ with an unbroken group $E_7 \times \mathrm{SU}(2)_{KK} \times \mathrm{U}(1)_R$. However, combined with other monopoles in the manner described above, one can break the group down to $\mathrm{SO}(10) \times \mathrm{SU}(2)_{KK}$. The number of families will nevertheless be still large.

Apart from the proliferation of the number of families and other shortcomings for a realistic model building with tree level considerations (such as the absence of a realistic Higgs spectrum and Yukawa couplings at tree level), there is a fundamental difficulty with all such compactifications. In fact it seems that, with the exception of the simplest compactifying solution in which the monopole is embedded in the $\mathrm{U}(1)_R$ factor, all other compactifications are unstable. To see this, let us embed the monopole in one or both of the non-abelian factors. Denote by V one of the excitations of the vector potential tangent to $\mathbf{S}^2$ and charged with respect to $\mathrm{U}(1)_M$. This vector has the components $V_\pm$ with respect to a complex basis in the tangent space of $\mathbf{S}^2$. We also have the reality condition $V = V^\dagger$. As a Lie-algebra-valued vector we can write V as $V = U_+^r T_r + W_+^r T_r^\dagger$, where U and W are complex fields and the T's are among the charged generators of the gauge group. For example they can be the generators of E_7 in the directions of $\mathbf{10}_2$ or $\mathbf{16}_1$ of the previous section. In order to be able to write down a general formula which can be applied for any model of this kind, denote the $\mathrm{U}(1)_M$ charge of U or W by q. The mass spectrum of $D = 4$ spin-zero fields resulting from such a $D = 6$ object is given by [22]

$$a^2 M^2 = \ell(\ell + 1) - (\lambda - 1)^2 \,, \tag{5.5}$$

where $\ell = |\lambda|, |\lambda| + 1, \ldots$ and $\lambda = 1 + \frac{n}{2} q$. It is easy to see that for all those fields for which $nq \leq -2$ there is a tachyon. For example, with positive n, the leading mode in the spectrum of $\mathbf{10}_2$ will be a tachyon of squared mass $-n/2a^2$.

The only way to avoid this conclusion is to find an embedding for which $|nq| = 1$ for all the excitations. With integer n and q we then need to have $n = \pm 1$ and $q = \pm 1$ for all the fields. Such an embedding is guaranteed to exist in all cases where there is a gauge group factor G_A that has a maximal-subgroup decomposition $G_A \supset H \times \mathrm{U}(1)$ with H simple and where all fermions charged under this group transform in the adjoint. This is exactly what happens in the $E_7 \times E_6 \times \mathrm{U}(1)_R$ model, where the monopole can be embedded in the "hidden" E_6 that gives rise to $\mathbf{16}$'s of SO(10) with $q = \pm 1$. The monopole with the minimal charge of ± 1 gives thus a stable compactification with two chiral families of SO(10) in the 16-dimensional representation transforming as singlets under the unbroken $E_7 \times \mathrm{SU}(2)_{KK}$ but charged relative to the unbroken $\mathrm{U}(1)_R$. It seems difficult to obtain an analogous result for the new model presented in this paper essentially because any embedding which produces integer q's necessarily contains fields for which $|q| \geq 2$. The only exception is of course the half-supersymmetric solution in which the monopole is embedded in $\mathrm{U}(1)_R$. This embedding will leave $E_7 \times G_2$ unbroken.

6. Conclusions

In this paper, we have demonstrated the existence of a second consistent $\mathcal{N} = 1$ gauged supergravity model in six dimensions besides the old $E_7 \times E_6 \times \mathrm{U}(1)_R$ model. The model found here is based on the $E_7 \times G_2 \times \mathrm{U}(1)_R$ gauge group, with hypermatter transforming as a half-hypermultiplet in the pseudoreal representation $(\mathbf{56}, \mathbf{14})$. The theory satisfies a set of very stringent anomaly constraints and turns out to be free of local and global anomalies. Also, as its $E_7 \times E_6 \times \mathrm{U}(1)_R$ sibling or, in fact, any other model of this type, the theory admits a monopole compactification on $\mathbb{R}^4 \times \mathbf{S}^2$. Despite the fact that embedding monopole type configurations in $E_7 \times G_2$ will produce many chiral fermions in $D = 4$, it seems that all such solutions are perturbatively unstable. On the other hand, if the monopole is identified with the vector potential of $\mathrm{U}(1)_R$ only, a stable compactification will be obtained and, with the choice of $n = 1$, half of the $D = 6$ supersymmetries will be unbroken. The model will clearly inherit all the brane solutions discovered so far in the context of $\mathcal{N} = 1$ supergravity models in six dimensions, to some of which we referred in section 1.

As far as the tachyons are concerned, one may adopt the point of view that they are welcome in the context of an effective theory as they are natural candidates for $D = 4$ Higgs fields. The quartic term in the potential for such fields will come from the self-coupling of the $D = 6$ gauge fields and their *vev* will break the $E_6 \times \mathrm{SU}(2)_{KK}$ (or $\mathrm{SO}(10) \times \mathrm{SU}(2)_{KK}$) at the KK scale. Such an origin for the Higgs fields has been considered before as a possible solution to the hierarchy problem. In order for this interpretation to be complete, one needs to look for new stable solutions of the six-dimensional field equations which would correspond to the minimum of the potential for the tachyons interpreted as Higgs fields. These solutions will necessarily break the spherical symmetry and their construction may give a geometrical origin to the Higgs mechanism. It will be interesting to find such solutions.

On the other hand, for phenomenological reasons, we may want to prevent certain modes from becoming tachyonic. In this paper we gave a necessary and sufficient condition for this to happen. Namely, in order for an excitation of an internal component of the gauge field not to be tachyonic, it is sufficient that $|nq| = 1$, where nq is understood as the sum of the individual nq's over all the monopole directions with respect to which the corresponding excitation is charged.

A final question, motivated by the uniqueness of the $D = 6$ gauged supergravities under consideration and the fact that they cannot be constructed through straightforward string or M-theory compactifications, refers to their origin in terms of a higher-dimensional fundamental theory. Although previous experience might suggest that such models possibly arise due to some new mechanism involving non-perturbative physics, such a mechanism has not been identified up to date.

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A. Anomaly polynomials

The anomaly structure of $D = 6$ theories is encoded in a set of formal eight-forms, called anomaly polynomials. In our conventions, the gravitational anomaly polynomials [3] are given by

$$
\begin{aligned}
I_8^{1/2}(R) &= \frac{1}{360}\,\mathrm{tr}\,R^4 + \frac{1}{288}(\mathrm{tr}\,R^2)^2\,, \\
I_8^{3/2}(R) &= \frac{49}{72}\,\mathrm{tr}\,R^4 - \frac{43}{288}(\mathrm{tr}\,R^2)^2\,, \\
I_8^{A}(R) &= \frac{7}{90}\,\mathrm{tr}\,R^4 - \frac{1}{36}(\mathrm{tr}\,R^2)^2\,.
\end{aligned} \tag{A.1}
$$

The gauge anomaly polynomials are given by

$$
\begin{aligned}
I_8^{1/2}(F) &= \frac{2}{3}\,\mathrm{tr}\,F^4\,, \\
I_8^{1/2}(F_A, F_B) &= 4\,\mathrm{tr}\,F_A^2\,\mathrm{tr}\,F_B^2, \\
I_8^{3/2}(F) &= \frac{10}{3}\,\mathrm{tr}\,F^4\,,
\end{aligned} \tag{A.2}
$$

where the second polynomial applies to the case where product representations are present. Finally, the polynomials corresponding to mixed anomalies are

$$
\begin{aligned}
I_8^{1/2}(F,R) &= -\frac{1}{6}\,\mathrm{tr}\,R^2\,\mathrm{tr}\,F^2\,, \\
I_8^{3/2}(F,R) &= \frac{19}{6}\,\mathrm{tr}\,R^2\,\mathrm{tr}\,F^2\,.
\end{aligned} \tag{A.3}
$$

Here, the superscripts 1/2, 3/2 and A refer to a spin 1/2 fermion, a spin 3/2 fermion and a 2-form potential respectively. The above anomaly polynomials correspond to Weyl spinors of positive chirality and 2-form potentials with self-dual field strengths. For negative-chirality spinors or anti-self-dual field strengths, the sign of the anomaly is reversed.

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